國立中正大學機械工程學系

九十一年度博士班入學考試試題卷

工程數學

科目總分：100分

考試時間：2個小時

中華民國九十一年六月十日

第一大題

1. It is well known that a radioactive substance decompose at a rate proportional to the amount of the substance. If at time = 0 sec, the mass of a radioactive substance is 4 grams and the proportional constant is $-1.2\times10^{-10}\,\text{sec}^{-1}$. (a) Please write down the ordinary differential equation describing the rate of decomposition of the radioactive substance (5 %). (b) Please solve the ordinary differential equation and determine the mass of the radioactive substance at time = 10^{10} sec. (10%)

2. Determine the steady-state output of the following ordinary differential equation by means of the complex method.(10%)

$$y'' + 2y' + 2y = \cos 2t$$

第二大題

Recall that for an n-dimensional matrix operator A the characteristic equation is of the polynomial form

$$\det\left(A - \lambda I\right) = (-1)^n \lambda^n + a_1\lambda^{n-1} + \cdots + a_n = 0.$$

According to the *Fundamental Theorem Algebra*, this equation must have at least one root, and so there must exist at least one eigenvalue. For the infinite-dimensional case, this situation is not necessarily true.

(a) Show that

$$y'' + \lambda y = 0, \qquad 2y(0) - y(1) + 4y'(1) = 0, \quad y(0) + 2y'(1) = 0$$

has no eigenvalues. **(13%)**

(b) By contrast, show that for

$$y'' + \lambda y = 0, \qquad y(0) - y(1) = 0, \quad y'(0) + y'(1) = 0$$

every λ (real or complex) is an eigenvalue! **(12%)**

第三大題

Given the partial differential equation with the boundary conditions as:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

Initial condition $u(x,0) = \sin x, \frac{\partial u}{\partial t}(x,0) = 0$ when $0 \le x \le \pi$

Boundary condition: $u(0,t) = 0, u(\pi,t) = 0$, when $t>0$

(a) Please list any least on physical system which can be described by the PDE equation mentioned above. Please be specific.(5%)
(b) Determine the u(x,t) by solving the PDE equation with the given conditions.(13%)
(c) If the PDE equation is changed to a non-homogenous equation given as:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \sin x$$

Initial condition: $u(x,0) = \sin x, \frac{\partial u}{\partial t}(x,0) = 0$ when $0 \le x \le \pi$

Boundary condition: $u(0,t) = 0, u(\pi,t) = 0$, when $t>0$

Please show the steps in determining the u(x,t) ? You don't need to solve the non-homogenous equation completely, just specify the necessary changes as compared to solving the problem in part (b). (7%)

第四大題

1. $f(z) = z^3$, where z is a complex number. Please use Cauchy-Riemann equations to show that $f(z)$ is analytic. (5%)

2. The function $f(z) = z^{-4} \sin z$ is integrated counterclockwise around the unit circle C. (a) Find the Laurent series of $f(z)$ (5%), (b) Find the result of the integration (5%).

3. Find the answer of the following integration: (10%)

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$$