1. Solve the following differential equations.

   (a) $x^2y' = y^2 + xy$. (10%)

   (b) $y'' + y = -3\sin 2x$. (15%)

2. The velocity vector field for an ideal flow is given by

$$\vec{V} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$$

   Determine the constants ***a***, ***b***, and ***c*** so that $\vec{V}$ is irrotational flow. (5%)

3. Evaluate $\oint_C (x + y^2)dx + (2x^2 - y)dy$ by

   (a) Using line integral,

   (b) Using Green's Theorem,

   where C is the boundary of the region determined by the graphs of

   $y = x^2$, $y = 4$. (20%)

4. A harmonic function:

$$f(\theta) = a\cos\theta + b\sin\theta$$

can be expressed as

$$f(\theta) = r\cos(\theta + \alpha),$$

where r and θ stand for its magnitude and phase, respectively.

(a) Derive (r, α) in terms of a given pair (a, b). (10%)

(b) The periodically binary signal u with period $T = 0.01\text{sec}$, as shown in Figure Prob. #4, can be expanded as Fourier series as

$$u(t) = \sum_{k=1}^{\infty} r_k \cos(\omega_k t + \alpha_k).$$

Calculate $(\omega_k, r_k, \alpha_k)$ above for $k = 1,2,\ldots$ (15%)

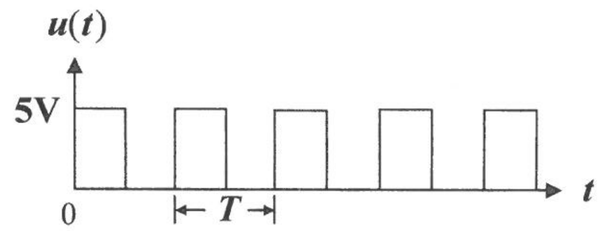

Figure Prob. #4

5. (a) Consider a matrix $A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

(1) Find the eigenvalues. (4%)

(2) Diagonalize the matrix A. (6%)

(b) Find the inverse matrix of the following matrix $B = \begin{bmatrix} -2 & 4 & 1 \\ 6 & 3 & -3 \\ 2 & 9 & -5 \end{bmatrix}$. (15%)