1. (20%) The two balls are attached to the light rigid rod, which is suspended by a cord from the support above it. If the balls and rod, initially at rest, are struck by the force F=100N, calculate the corresponding acceleration a of the mass center.

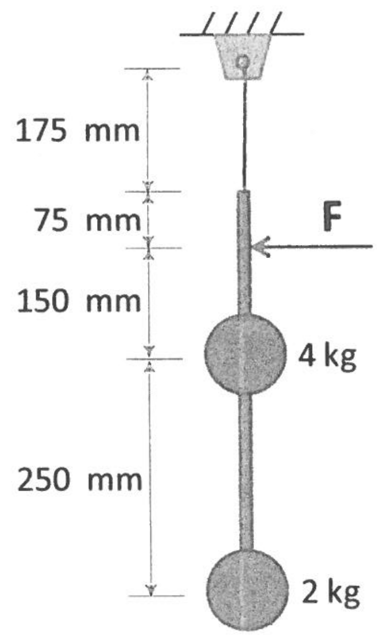

Fig. 1

2. (20%) The 100-kg crate is projected along the floor with an initial speed of 8 m/s at x=0. The coefficient of kinetic friction is 0.4. Calculate the time required for the crate to come to rest and the corresponding distance x traveled.

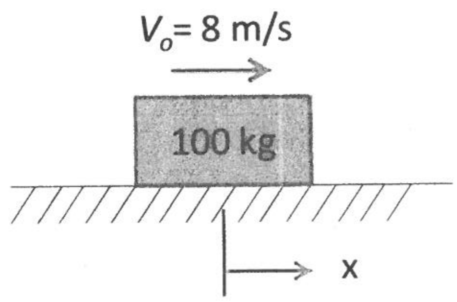

Fig. 2

3. (10%) The polar coordinates of a particle are given by $r = 2 + 5t^2/2$ and $\theta = 0.5 + 3t^2/4$ where r is in millimeters, θ is in radians, and t is in seconds. Determine the magnitude of the velocity **v** and the r- and θ- components of the acceleration **a** of the particle when $t = 3$ s.

4. (30%) Consider a system consisted of two rigid bodies, denoted as elements A and B, as shown in Fig. 3. Element A is a rectangular plate which can rotate freely with respect to element B. Element B is a rectangular plate with a motor (that drives element A). It is connected to a fixed wall and can rotate freely with respect to a fixed axis. Assume that all contacts are frictionless. Suppose that there are two body-fixed coordinates $X_AY_AZ_A$ and $X_BY_BZ_B$ for elements A and B, respectively. These coordinates are located at their mass centers and coincide with their principal axes. The mass centers of elements A and B are on the same line (i.e., the rotation axis of element A), and the distance is l. Let us denote the following notations.

m_A, m_B : mass of elements A and B, respectively.

I_{AX}, I_{AY}, I_{AZ} : mass moment of inertia of element A with respect to the 3 coordinate axes X_A, Y_A, Z_A (which are also principal axes).

I_{BX}, I_{BY}, I_{BZ} : mass moment of inertia of element B with respect to the 3 coordinate axes X_B, Y_B, Z_B (which are also principal axes).

$\theta_A, \dot{\theta}_A$: angular displacement and velocity of element A with respect to its rotation axis (denoted by the dash-dot line in Fig. 3).

$\theta_B, \dot{\theta}_B$: angular displacement and velocity of element B with respect to its rotation axis (denoted by the dashed line in Fig. 3).

g : gravitational acceleration.

l: distance between the mass centers of elements A and B.

(a) (20%) Please express the kinetic energy and potential energy of element A in terms of the given variables.

(b) (10%) Please express the kinetic energy and potential energy of element B in terms of the given variables.

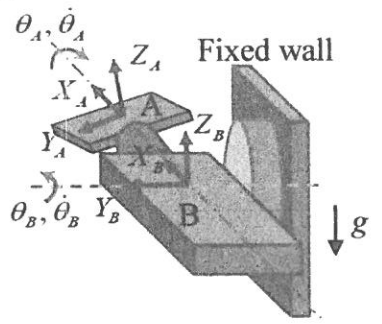

Fig. 3

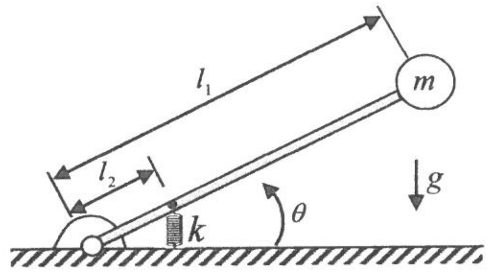

Fig. 4

5. (20%) Consider the vibration system shown in Fig. 4, where a point mass m is connected to the ground with a massless rigid link of length l_1 and angle θ. A spring is placed between the link and the ground as shown in the figure. Assume the rotating joint is frictionless. Suppose that: $m = 0.1\text{ kg}, l_1 = 1\text{ m}, l_2 = 0.2\text{ m}, k = 100\text{ N/m}, g = 10\text{ m/s}^2$. Suppose also that the un-stretched length of the spring is $h_0 = 0.15\text{ m}$.

(a) (5%) When $\theta = \theta^*$, the static equilibrium condition is reached. Please find θ^*.

(b) (15%) Please find the natural frequency of the system near the equilibrium position (i.e., $|\theta - \theta^*|$ is small). Note: You can take the following approximations: $\cos\theta = \cos\theta^*$ and $\sin\theta = \sin\theta^* + (\theta - \theta^*)\cos\theta^*$.