1. Given a differential equation, $x \cdot dy = \left[x \cdot \cos(3x) - 2y \right] \cdot dx$

(a) **(15%)** Solve the differential equation in terms of explicit function, $y = f(x)$.

(b) **(10%)** Find the tangent line equation at $x = \pi/3$ and $y = 1/(3\pi)$ for the above $y = f(x)$.

2. (a) Calculate the Laplace transform of the following functions. Do not simply write your answer. You have to show how you find the transform.

I. **(5%)** $f(t) = \sin(10t)$

II. **(5%)** $f(t) = \cosh(t)\sin(10t)$

(b) **(5%)** Use Laplace transform to solve the following ordinary differential equation.

$$\frac{d^2y}{dt^2} + 25y = \cos(5t), \quad \frac{dy(0)}{dt} = 0, \quad y(0) = 0$$

(c) About Fourier series expansion.

I. **(5%)** Write down the Fourier series expansion formula of a periodic function $f(t)$ with a period 2π.

II. **(5%)** Determine the Fourier series representation of the periodic function $f(t) = e^t$ for $-\pi < t < \pi$, and $f(t + 2\pi) = f(t)$.

3. Let $\mathbf{A} = \begin{bmatrix} 5 & 10 & -10 \\ 10 & 5 & -20 \\ 5 & -5 & -10 \end{bmatrix}$.

(a) **(10%)** Find the eigenvaules and eigenvectors of $\mathbf{A}$.

(b) **(7%)** Find a square matrix $\mathbf{X}$ so that $\mathbf{X}^{-1}\mathbf{A}\mathbf{X}$ is a diagonal matrix.

(c) **(8%)** Evaluate e^{At}.

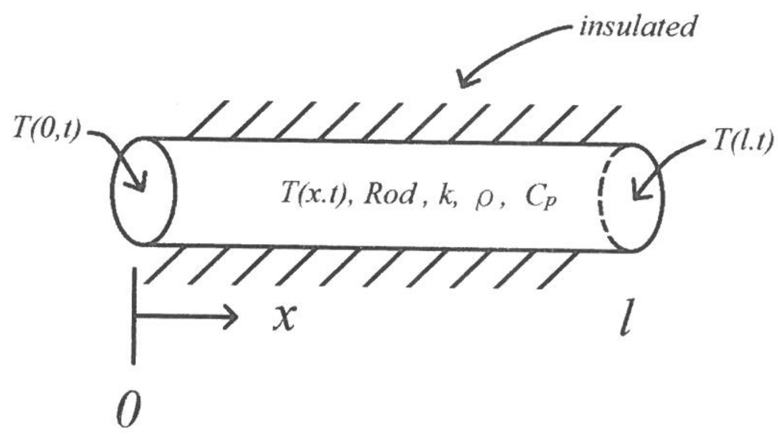
(Do not simply write your answer; show the details of your work.)

4. (a) I. **(5%)** Write down the physical meaning for one-dimensional unsteady heat conduction.

II. **(10%)** For the rod with a constant cross-section area and a length of l, as shown in the figure, find the temperature distribution function of $T(x,t)$ for the rod by solving the diffusion equation,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial T}{\partial t}$$

where T is the temperature, x is the position, t is the time, $\alpha = \frac{k}{\rho \times C_p}$ is a constant with k being the thermal conductivity, ρ being the density, and C_p being the specific heat of the rod. The boundary and initial conditions are $T(0,t)=0$, $T(l,t)=0$, and $T(x,0)=f(x)\neq 0$.



(b) I. **(3%)** Explain analytic function $f(z)$ in domain D.

II. **(7%)** Let $f(z)$ be analytic function in domain D, and Let C be a simple closed contour lying entirely within D. If z_0 is a point within C, then prove that

$$f(z_0) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z - z_0}\,dz$$