

1. (25%) Derive Laplace Transforms of the following functions u of time t :

(a) (5%) $u(t) = \delta(t - \tau)$, $\tau > 0$;

(b) (5%) $u(t) = e^{st}1(t)$, s is a complex number;

(c) (5%) $u(t) = \cos(\omega t - \tau)1(t - \tau)$, $\omega, \tau > 0$;

(d) (5%) $l(t) = u(t) + \int_0^t u(\tau) d\tau$, and

(e) (5%) $u(t) = \int_0^t \sin(t - \tau) d\tau$, $\tau > 0$.

where δ and 1 denote the unit-pulse and unit-step function of time, respectively.

2. (25%) Consider the following dynamics G with the transfer function being

$$G(s) \equiv \frac{11s^2 + 11s + 10}{(s + 10)(s^2 + s + 1)}.$$

(a) (5%) Is G asymptotically stable? Is it bounded-input-bounded-output stable?

Is it exponentially stable?

(b) (5%) What is the steady-state error of the unit-step response?

(c) (5%) Estimate the rising time of the unit-step response.

(d) (5%) Estimate the overshoot of the unit-step response.

(e) (5%) Estimate the settling time of the unit-step response.

3. (25%) Given the unit-feedback system shown in Figure 1 with the open loop transfer function $G(s)$ as:

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

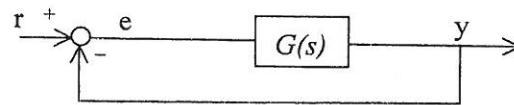


Figure 1

- (5%) Determine the closed loop transfer function
 - (5%) Determine the range of K which makes the closed loop system stable
 - (10%) Plot the root locus and determine the breakaway points and asymptotes.
 - (5%) Find the locations at the $j\omega$ axis (imaginary axis) when the root locus cross the imaginary axis.
4. (25%) Consider the closed loop system as shown in Figure 2.

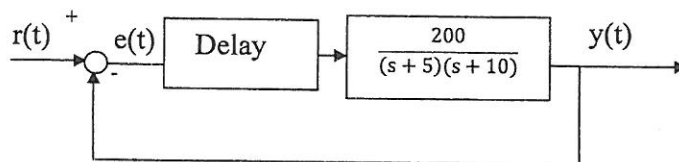


Figure 2

- (5%) Plot the Bode plot of open loop system in details when the time delay is equal to 0.
- (10%) Plot the Nyquist diagram of $G(s)$, and determine the gain margin and phase margin of the system when the delay is equal to 0.
- (5%) For the time delay time equal to 0.1 second, determine the gain margin.
- (5%) Determine the range of the delay time approximately which makes the system become unstable. Explain the delay effect on the stability of the closed loop system.