1. It is known that the variation of pressure, $\triangle p$, within a static fluid is dependent upon the specific weight of the fluid, γ, and the elevation difference, $\triangle z$. Using dimensionless analysis find the form of the hydrostatic equation for pressure variation. (10%)

2. It is known that the velocity potential for a rotating cylinder (with a radius, a) rotating in a uniform stream of fluid can be obtained by the summation of the velocity potentials for a uniform flow, a doublet and a free vortex.
   (a) Please show that the velocity potential for such a flow is

   $$\phi = Ur(1+\frac{a^2}{r^2})\cos\theta + \frac{\Gamma}{2\pi}\theta$$

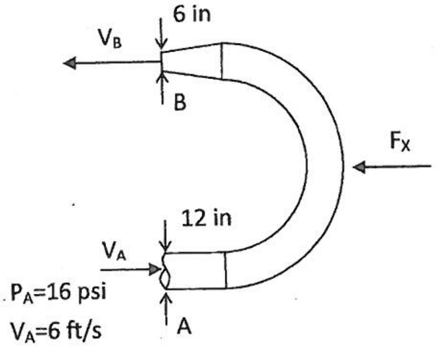
   where U, a, and Γ are the velocity of the uniform stream, radius of the cylinder and circulation, respectively. (10%)
   (b) Assuming the flow is flowing from the left to the right, please determine the value of the circulation when the stagnation point is located at the point on the cylinder (1) facing the flow and (2) at the bottom. (10%)

3. A laminar boundary layer velocity profile is approximated by

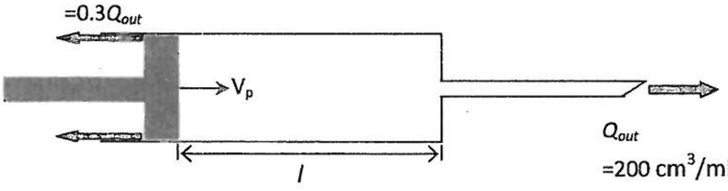
   $$u/U = [2-(y/\delta)](y/\delta) \text{ for } y \le \delta \text{ and}$$
   $$u = U \text{ for } y > \delta$$

   (a) Show that the profile satisfies the appropriate boundary conditions. (10%)
   (b) Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$. (10%)

4. Water flows through a horizontal, 180° pipe and nozzle system as shown in the figure.
   (a) Please calculate V_B=? [10%]
   (b) Please find the head loss from A to B. [10%]
   (c) Please calculate the force required to hold the system in place. [15%]
   (1 psi=144 lb/ft^2, g=32.2 ft/s^2, Specific weight for water is 62.4 lb/ft^3)



5. A syringe is shown in the figure. The plunger has a face area of 500 mm^2. If the liquid in the syringe is to be injected steadily at a rate of Q_{out}=200 cm^3/min, at what speed V_P should the plunger be advanced? [15%]
   The leakage rate past the plunger is 0.3 times the volume flow rate out of the needle.



Summary of Basic, Plane Potential Flows.

Description of Flow Field	Velocity Potential	Stream Function	Velocity Components[a]
Uniform flow at angle α with the x axis	$\phi = U(x\cos\alpha + y\sin\alpha)$	$\psi = U(y\cos\alpha - x\sin\alpha)$	$u = U\cos\alpha$ $v = U\sin\alpha$
Source or sink $m > 0$ source $m < 0$ sink	$\phi = \dfrac{m}{2\pi}\ln r$	$\psi = \dfrac{m}{2\pi}\theta$	$v_r = \dfrac{m}{2\pi r}$ $v_\theta = 0$
Free vortex $\Gamma > 0$ counterclockwise motion $\Gamma < 0$ clockwise motion	$\phi = \dfrac{\Gamma}{2\pi}\theta$	$\psi = -\dfrac{\Gamma}{2\pi}\ln r$	$v_r = 0$ $v_\theta = \dfrac{\Gamma}{2\pi r}$
Doublet	$\phi = \dfrac{K\cos\theta}{r}$	$\psi = -\dfrac{K\sin\theta}{r}$	$v_r = -\dfrac{K\cos\theta}{r^2}$ $v_\theta = -\dfrac{K\sin\theta}{r^2}$

[a]Velocity components are related to the velocity potential and stream function through the relationships:
$u = \dfrac{\partial\phi}{\partial x} = \dfrac{\partial\psi}{\partial y}$　$v = \dfrac{\partial\phi}{\partial y} = -\dfrac{\partial\psi}{\partial x}$　$v_r = \dfrac{\partial\phi}{\partial r} = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$　$v_\theta = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta} = -\dfrac{\partial\psi}{\partial r}$