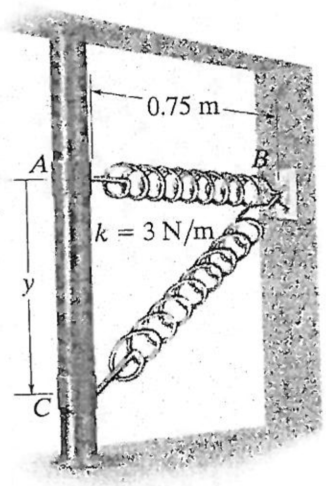
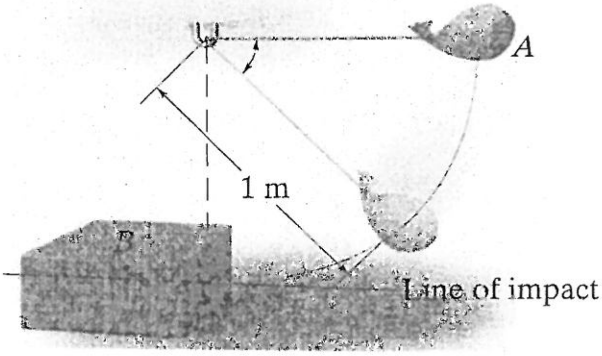
1. (20%) A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine
   (a) (7%) the distance traveled in 3.5 s,
   (b) (7%) the particle's average velocity during the time interval, and
   (c) (6%) the particle's average speed during the time interval.

2. (20%) A smooth 2-kg collar C, as shown below, fits loosely on the vertical shaft. If the spring is un-stretched when the collar is in the position A, determine the speed at which the collar is moving when y=1 m, if
   (a) (10%) it is released from rest at A, and
   (b) (10%) it is released at A with an upward velocity v_A=2 m/s.



3. (20%) The bag A, having a weight of 6 kg, is released from rest at the position $\theta = 0^\circ$, as shown below. After falling to $\theta = 90^\circ$, it strikes an 18-kg box B. If the coefficient of restitution between the bag and box is e=0.5, determine
   (a) (10 %) the velocities of the bag and box just after impact, and
   (b) (10%) the loss of energy during collision.



4. (30%) Consider a robot arm sitting on a stationary stand S through joint A as shown below where respectively, L_{AB}, L_{BC}, and L_{CD} denote the lengths of the arms AB, BC, and CD, and θ_A and θ_B denote the angles of the arms AB and BC.
Assume the arm AB has a constant clockwise angular velocity ω_{AB}, the arm BC has a constant clockwise angular velocity ω_{BC}, and the arm CD remains vertical. Note the angular velocities are constant; hence, the angular accelerations are zero.

(a) (5%) Discuss how the following velocity expression of the velocity of point B $\vec{v}_B$ was obtained:

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

where $\vec{v}_A$ and $\vec{r}_{B/A}$ respectively represent the velocity of point A and relative position vector of B with respect to A.

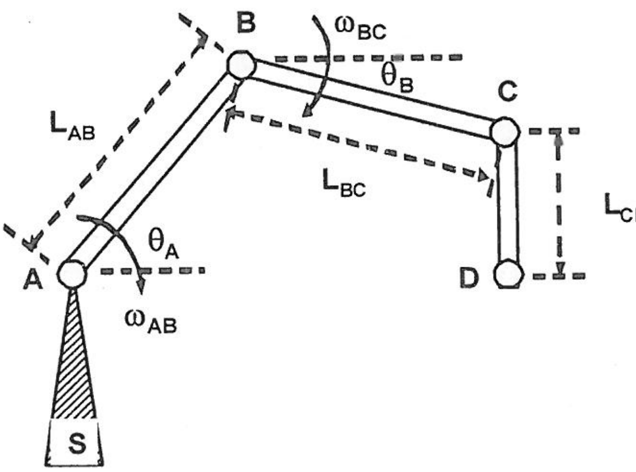
(b) (5%) Take the time derivative of the expression in (a) to derive the following expression of the acceleration of point B $\vec{a}_B$:

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{AB} \times \left(\vec{\omega}_{AB} \times \vec{r}_{B/A}\right)$$

where $\vec{a}_A$ and $\vec{\alpha}_{AB}$ respectively represent the acceleration of point A and angular acceleration vector of the arm AB.

(c) (10%) Let $L_{AB} = 200$ mm, $L_{BC} = 200$ mm, $L_{CD} = 120$ mm, $\theta_A = 60^\circ$, $\theta_B = 30^\circ$, $\omega_{AB} = 1.0$ rad/s, and $\omega_{BC} = 0.1$ rad/s. Use the expression in (a) to calculate the velocity of points C and D.

(d) (10%) With the same numerical values of arm lengths, angles, and angular velocity, use the expression in (b) to calculate the acceleration of points C and D.



5. (10%) Consider a sliding-contact linkage fixed to the ground as shown below where respectively, L_{AB} and L_{BC} denote the lengths of the arms AB and BC and θ_A and θ_C denote the angles of the arms AB and CB.

(a) (5%) Derive the following velocity expression of the velocity of point B $\vec{v}_B$:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B,rel} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

where $\vec{v}_A$, $\vec{\omega}_{AB}$ and $\vec{r}_{B/A}$ respectively represent the velocity of point A, the angular velocity of the arm AB, and the relative position vector of B with respect to A, and in particular, $\vec{v}_{B,rel}$ denotes the velocity of point B relative to a body coordinate system fixed to the arm AB.

(b) (5%) Assume the angular velocity of the arm AB is counterclockwise. Express the angular velocity $\vec{\omega}_{BC}$ in terms of L_{AB}, L_{BC}, θ_A and θ_C.