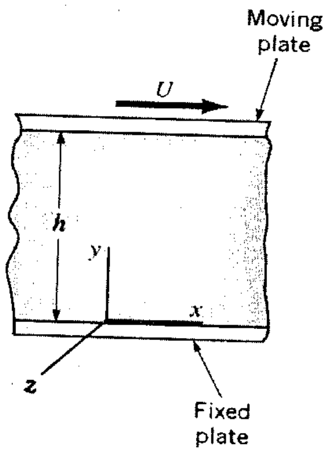
1. Two horizontal, infinite, parallel plates are spaced a distance h apart. A viscous liquid is contained between the plates. The bottom plate is fixed and the upper plate moves parallel to the bottom place with a velocity U, as shown in the following figure. The liquid motion is caused by the liquid being dragged along by the moving boundary. There is no pressure gradient in the direction of flow.



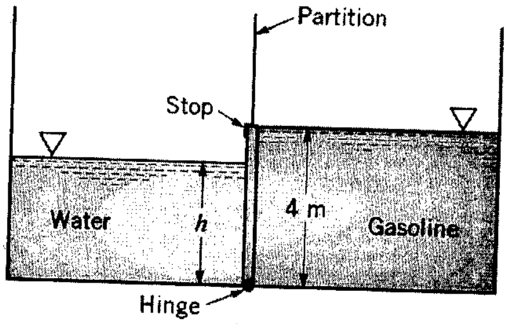
(a) Assume the laminar and steady flow. Start with the Navier-Stokes equations, and reduce them to find the proper differential equation to satisfy this problem. **(5%)**

(b) Please specify proper boundary conditions. **(6%)**

(c) Determine the velocity distribution between the plates. **(5%)**

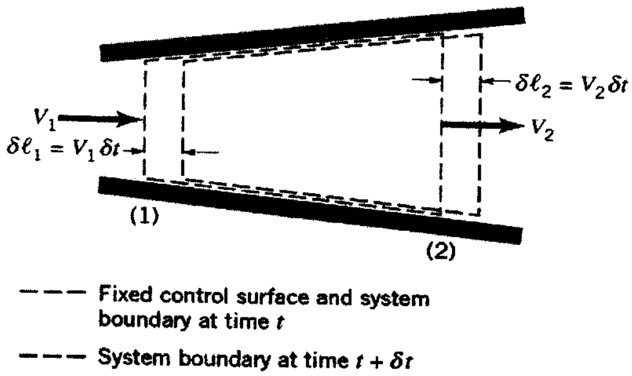
(d) Determine an expression for the flowrate passing between the plates (for a unit width). Express your answer in terms of h and U. **(4%)**

2. An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ \text{kg/m}^3$ at a depth of 4 *m*, as shown in the following figure. A rectangular gate that is 4 *m* high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, *h*, will the gate start to open? **(10%)**



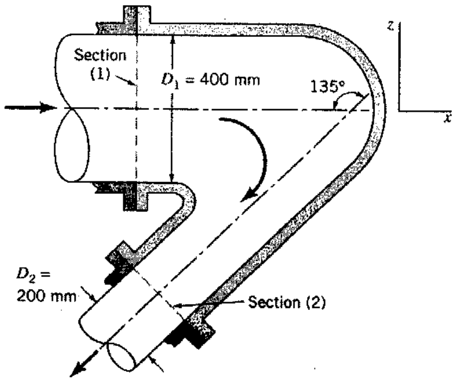
3. Using the following figure, please derive the Reynolds transport theorem given below. **(12%)**

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 b_2 A_2 v_2 - \rho_1 b_1 A_1 v_1$$



4. Using the Reynolds transport theorem given above (in Problem 3), please derive the mass conservation law for a finite control volume **(3%)**

5. A converging elbow (shown below) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at section (1) and 200 mm, section (2). The elbow flow passage volume is 0.2 m^3 between sections (1) and (2). The water volume flow rate is 0.5 m^3/s and the elbow inlet and outlet pressures are 200 kPa and 100 kPa, respectively. The elbow mass is 12 kg. Calculate the velocities at sections (1) and (2) (**4%**), and the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place (**16%**).



6. The Navier-Stokes equation, an equation of motion for Newtonian fluids of constant density and viscosity, can be expression as:

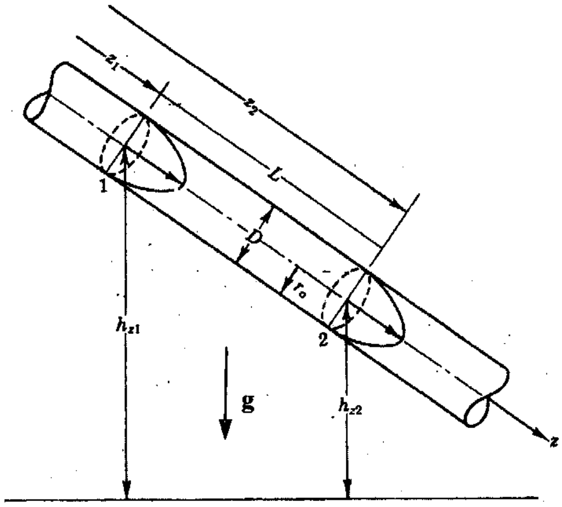
$$\rho \frac{D\mathbf{V}}{Dt} = \rho\mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V} \tag{1}$$

In Eq. (1): ρ and μ represent the fluid density and viscosity, respectively; V and g denote respectively the vectors for velocity and acceleration of gravity; $\frac{D}{Dt}$ represents the derivative following the fluid (or the substantive derivative); and, the symbol ∇ (read "del") is the gradient operator which is a vector quantity. Let L, U and P represent respectively the characteristic reference magnitudes of length, velocity, and pressure. If X^*, Y^*, Z^*, P^*, and V^* are dimensionless variables for x, y, z, p, and V, respectively, and ∇^* is the dimensionless symbol for the operator ∇.

(a) Find the dimensionless form of equation for Eq. (1). (**10%**)

(b) For the dimensionless groups shown in (a), do you know their official names and their physical meanings? (**15%**)

7. Driven by the gravity, an incompressible viscous flow with constant fluid properties is flowing in a straight pipe with constant cross section, as shown in the attached figure. Referring to the governing equations shown below, find the steady-state velocity distribution in the pipe. **(10%)**



Governing equations for the cylindrical coordinate (r, θ, z):

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)+\frac{\partial}{\partial z}(v_z)=0$$

The r - momentum equation:

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}+g_r$$

The θ - momentum equation:

$$\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]-\frac{1}{\rho r}\frac{\partial p}{\partial\theta}+g_\theta$$

The z - momentum equation:

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_z}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_z)\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z$$