1. Please find the inverse of the given Laplace transform below. (10%)

$$F(s) = \frac{1}{s^2 + 4s + 5}$$

2. (a) Determine whether the function $g(x) = x\sin(x)$ is even, odd, or neither. (5%)

   (b) Please find the Fourier series of the periodic function: (10%)

   $F(x) = x, \; |x| < T.$

   (*Hint: Fourier cosine series* -- $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi x}{T}$, *Fourier sine series* -- $F(x) = \sum_{n=1}^{\infty} b_n \sin\frac{2n\pi x}{T}$)

3. The Euler-Cauchy equation of the third order is

   $x^3 y''' + ax^2 y'' + bxy' + cy = 0$

   Show that $y = x^m$ is a solution of the equation if and only if m is a root of the auxiliary equation. (15%)

   $m^3 + (a-3)m^2 + (b-a+2)m + c = 0$

4. Considering the heat flow in a long thin bar or wire of constant cross section and homogeneous material, which is insulated laterally. The heat flows in the x-direction only and is governed by the one-dimensional heat equation as shown below. Assume a rod with the length of L, is kept at 100 ℃ for a long time. Then at some instant, say, at t = 0, the temperature at x=L is suddenly changed to 0 ℃ and kept at this value, while the temperature at x=0 is kept at 100 ℃. Solve the temperature function u(x,t) in the rod. (20%)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where u(x,t) is the temperature in this body, c is the physical property.

5. A curve C in space is represented by a vector function, C(t) = [x(t), y(t), z(t)], where x, y, z are Cartesian Coordinates, and $x(t) = \frac{2}{3}t^3$, $y(t) = t^2 + 1$, $z(t) = t+2$.

(a) Determine the unit tangent vector at t = 1. (5%)

(b) Determine the curve length for $0 \le t \le 2$. (5%)

6. Given a matrix A, as shown below, find a 3×3 matrix M such that $\left(M^{-1} \cdot A \cdot M\right) = D$ is diagonal. Show the matrix M and the diagonal matrix D. (15%)

$$A = \begin{bmatrix} 1.5 & -0.5 & 2.5 \\ -0.5 & 1.5 & -0.5 \\ 0 & 0 & -1 \end{bmatrix}$$

7. (a) Let f(z) be analytic in a domain D and C be a simple closed curve in D, please describe the Cauchy's residue theorem to evaluate $\oint_c f(z)dz$. (7%)

(b) Using the theorem in (a) to evaluate $\oint_{|z|=2} \frac{e^z}{z(z^2-1)} dz$, please show all the details. (8%)