

1. (15%) Solve the following ODE:

$$y' = y \frac{(x-1)^2}{y+3}$$

2. (10%) Solve for the general solutions of the following ODEs:

(a) (5%)  $y'' - y' - 12y = 0$

(b) (5%)  $4y'' - 4y' + y = 0$ .

3. (25%) Consider the following 3 functions

$$f(x) = x$$

$$g(x) = 1 - \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi x$$

$$h(x) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin 2n\pi x$$

where  $g(x)$  and  $h(x)$  are the Fourier series expansions of the function  $f(x)$ , but are for  $f(x)$  restricted on different intervals. One interval is  $[0, 1]$ , and the other is  $[0, 2]$ .

- (a) (7%) Which function,  $g(x)$  or  $h(x)$ , is the Fourier series of  $f(x)$  on the interval  $[0, 1]$ ? State the reasons. **Note:** No score will be given without any reason.
- (b) (18%) Please find the function values of  $g(x)$  and  $h(x)$  at  $x = 0.8, 1.8$ , and  $3.8$ . That is, find

$$g(0.8), h(0.8), g(1.8), h(1.8), g(3.8), h(3.8).$$

4. (a) (15%) Find the eigenvalues and eigenvectors.

$$\begin{bmatrix} 8 & 0 & 3 \\ 2 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

- (b)(10%) Reduce the quadratic equation  $9x_1^2 - 6x_1y_1 + 17y_1^2 = 72$  to the canonical

form  $ax_2^2 + by_2^2 = c$ . Express  $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$  in terms of the new variables  $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$  and find

$a$ ,  $b$ , and  $c$ .

5. (25%) The velocity vector field for a flow is given by

$$\mathbf{V} = 3x^2y \mathbf{I} + (x^3+1) \mathbf{J} + 9z^2 \mathbf{K}.$$

- (a) (10%) Find the  $\text{div}(\mathbf{V})$ ,  $\text{curl}(\mathbf{V})$ .

- (b) (10%) If  $\mathbf{V} = \nabla\phi$ , where  $\phi(x,y,z)$  is a scalar function, try to find  $\phi(x,y,z)$ .

- (c) (5%) Find  $\int_C \mathbf{V} \cdot d\mathbf{r}$  where  $C$  is a straight line path from  $(0,0,0)$  to  $(1,1,1)$ .