1. An open tank has a vertical partition and on one side contains gasoline with a density ρ= 700 kg/m³ at a depth of 4 m, as shown in the figure. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open. (15%)

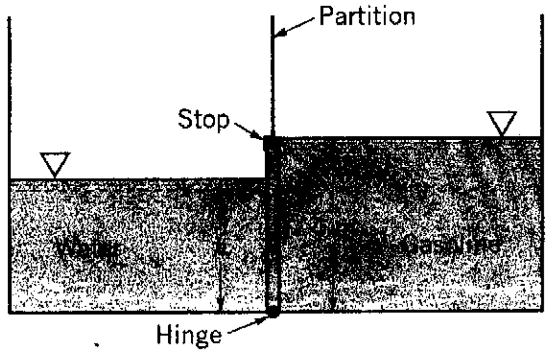


Figure 1

2. A velocity field for a non-viscous flow with constant density is given as:

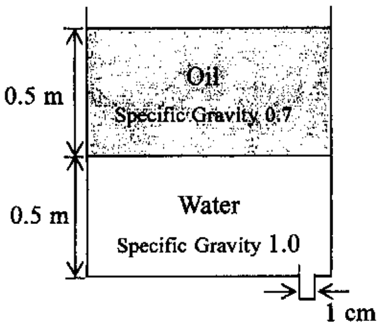
$$\vec{V} = \left[(y^2 - z^2 + 2xz - 2xy)\vec{i} + (z^2 - x^2 + 2xy - 2yz)\vec{j} + (x^2 - y^2 + 2yz - 2xz)\vec{k}\right] \text{ m/s}$$

(1) Determine whether this is a possible incompressible flow (5%)

(2) Determine whether this flow is irrotational (5%)

(3) Calculate the pressure change between points (0,0,1) and (1,1,2) (5%)

3. Water drain from the tank through a 1 cm diameter drain hole as shown in the figure. Please determine the flow rate of the drain hole. (20%) The viscous effects are negligible.

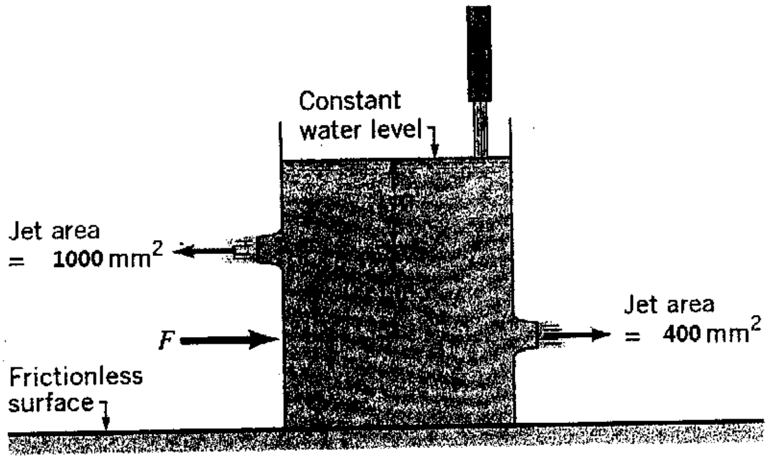


4. (1) Please explain the usage of Reynolds transport theorem given below.(3%)

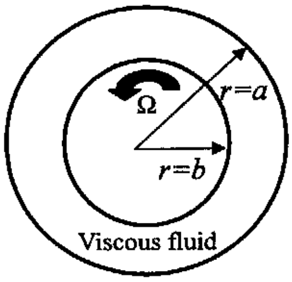
$$\frac{D}{Dt}\int_{sys}\rho d\forall = \frac{\partial}{\partial t}\int_{cv}\rho d\forall + \int_{cs}\rho\vec{v}\cdot\vec{n}dA$$

(2) Please use Reynolds transport theorem to derive the control volume expression for conservation of mass (law of mass conservation).(5%)

5. Water is added to the tank shown below through a vertical pipe to maintain a constant water level as shown in the figure. The tank is placed on a horizontal surface which has a frictionless surface. Assuming the water density is 1,000 kg/m³. Determine the water mass flow rate through the vertical pipe to maintain a constant water level (5%) and the horizontal force, F, required to hold the tank stationary.(12%) Neglect all losses in calculation.



6. A viscous fluid between two long cylinders as shown in the figure is set into motion by rotation counterclockwise of the inner cylinder at angular velocity Ω. The outer cylinder is fixed. Assuming steady flow in a two-dimensional cylindrical coordinate. Using governing equations shown in **Appendix A**, find the solutions for the following problems.
   (1) Show that $v_\theta = v_\theta(r)$ only. (5%)
   (2) List all expressions (including the boundary conditions) so that you can find the tangential velocity $v_\theta(r)$. Do not need to solve the equations you list. (10%)



Appendix A

The general governing equations for an incompressible flow in cylindrical coordinates (r,θ,z) are given as

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)+\frac{\partial}{\partial z}(v_z)=0$$

The r - momentum equation:

$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]-\frac{1}{\rho}\frac{\partial p}{\partial r}+g_r$$

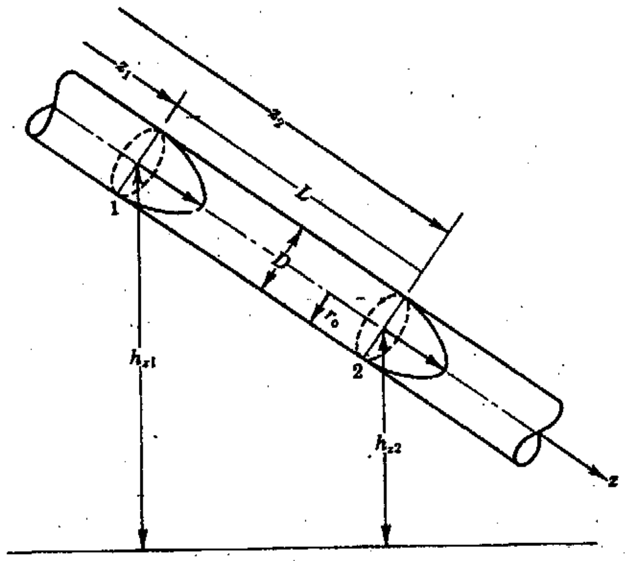
The θ - momentum equation:

$$\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}+g_\theta$$

The z - momentum equation:

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}=\frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_z)\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z$$

7. An axial flow of an incompressible fluid in a straight pipe of constant cross section, as shown in the figure, is at steady state. Starting from the general governing equations shown in **Appendix A**, show that $v_z = \frac{1}{4\mu}\left[-\frac{\partial}{\partial z}(\rho g h_z + p)\right]\left(\frac{D^2}{4} - r^2\right)$. (10%)



Appendix A

The general governing equations for an incompressible flow in cylindrical coordinates (r, θ, z) are given as

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

The r - momentum equation:

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z} = \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{1}{\rho}\frac{\partial p}{\partial r} + g_r$$

The θ - momentum equation:

$$\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z} = \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{\rho r}\frac{\partial p}{\partial \theta} + g_\theta$$

The z - momentum equation:

$$\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} = \frac{\mu}{\rho}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_z)\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{1}{\rho}\frac{\partial p}{\partial z} + g_z$$