1. (10%) If aerodynamic drag is taken into account, the acceleration of a falling object can be approximated by $a = g - cv^2$, where g is the acceleration due to gravity at sea level and c is a constant.

   1.a (5%) If an object is released from rest, what is its velocity as a function of the distance s from the point of release?

   1.b (5%) Show that when $c \to 0$, the limit of the solution of 1.a agrees with the solution you would obtain by assuming that $a = g$.

2. (10%) Consider the following potential function U(r) between two particles where r denotes the distance between them.

$$U(r) = \frac{-1}{r^6} + \frac{1}{r^{12}}$$

   2.a (5%) Plot the potential function by identifying the distance of minimum potential and analyzing the potential as $r \to \infty$ as well as $r \to 0$.

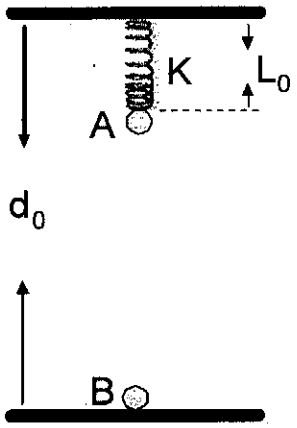
   2.b (5%) Determine the distance range over which the two particles repel each other.

3. (20%) Consider an experiment setup where particle A is fixed to the ceiling through a spring with a spring constant K and particle B is fixed to the ground as depicted in the following figure. Also noted in the figure are d_0= the distance between the ground and ceiling and L_0= the un-stretched length of the spring. Ignore the gravitational effect and assume the potential energy between the two particles is $U(r) = \frac{-1}{r^6}$. Let m_A denote the mass of the particle A.

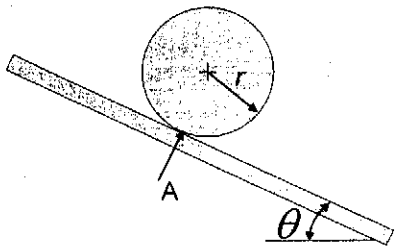
3.a (5%) Is the force determined by U(r) is conservative?

3.b (10%) Let x denote the elongation (伸長) amount of the spring from its un-stretched length. Write down the ordinary differential equation governing the motion of the particle A.

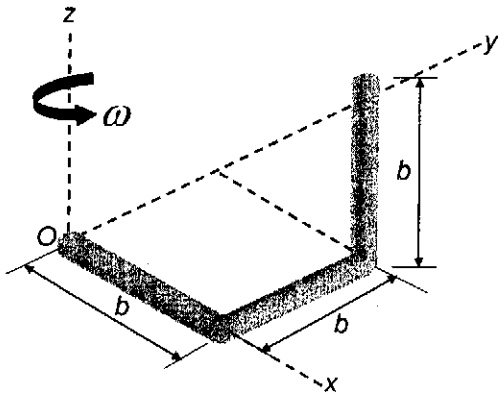
3.c (5%) Assume the elongation of the spring is very small, enough to justify linearization. Determine the equilibrium position of the particle A.

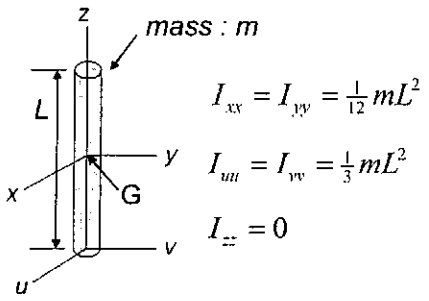


4. (15%) The ball of mass m and radius r rolls along an inclined plane for which the coefficient of static friction is μ (see the figure below). If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at A.



5. (15%) The whole bent rod has a mass M and revolves about the z-axis with an angular velocity ω (see the left figure below). Determine the angular momentum of the rod about the origin O of the coordinates for the position shown. Also find the kinetic energy of the rod. [*Hint: The right figure below shows the mass moments of inertia of a homogeneous slender rod*]





6. (30%) Regarding dynamics of a bike (see the figure below).

6.a (15%) Is the following statement correct?

"A bike with a smaller driving-wheel is able to move up a bigger slope."

Give a dynamics-based explanation for your answer.

6.b (15%) *"A bike with a smaller driving-wheel consumes its rider more energy."* Do you agree? Do a dynamical analysis to support your opinion.