1. (25%) Solve the following differential equations,

(a) (10%) $\dfrac{dy}{dx}=\dfrac{3x^3-2y}{x}$

(b) (15%) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y=e^{-2x}\cdot ln(x)$

2. (25%) Assume that **F**=$z$**i**+$x$**j**+$y$**k**, where **i**, **j**, **k** are three unit vectors in a Cartesian coordinate system. $C$ represents the curve of intersection of the plane $x+y+z=0$ and the sphere $x^2+y^2+z^2=1$.

(a) (8%) Determine the surface area enclosed by the curve $C$.

(b)(7%) Assume that the surface area enclosed by $C$ is oriented upward. Determine the unit vector normal to the area enclosed by $C$.

(c) (10%) Determine the line integral along the curve $C$; $\oint_C \mathbf{F}\bullet d\mathbf{r}$

Hint: 1. The area of an ellipse $x^2/a^2+y^2/b^2=1$ is $\pi ab$.

2. Stokes' Theorem $\oint_C \mathbf{F}\bullet d\mathbf{r}=\iint_S(\text{curl}\mathbf{F})\bullet \mathbf{n}dS$

3. (25%) Given the following ordinary differential equation $\ddot{y}+3\dot{y}+2y=f(t)$ with initial conditions given as $\dot{y}(0)=0, y(0)=0$.

(a) (10%) If $f(t)=4e^{-2t}$, please solve the equation using **Laplace Transform**.

(b) (7%) If $f(t)$ is a periodic function given as $f(t)=\begin{cases}0 & \text{for } 0<t<1\\ 1 & \text{for } 1<t<2\end{cases}$, $f(t+2)=f(t)$

Expand the function $f(t)$ by using the Fourier series.

(c) (8%) Solve the above ordinary differential equation if $f(t)$ is the periodical function given in (b).

4. (a) (3%) The partial differential equation is given by

$$A\frac{\partial^2 T}{\partial x^2}+B\frac{\partial^2 T}{\partial x\partial y}+C\frac{\partial^2 T}{\partial y^2}+D\frac{\partial T}{\partial x}+E\frac{\partial T}{\partial y}+FT=0$$

Find the conditions of **linear** second-order partial differential equation.

(b) (7%) Drive the 1-D Heat equation $k\frac{\partial^2 T(\mathrm{x},t)}{\partial \mathrm{x}^2}=\rho C\frac{\partial \mathrm{T}(\mathrm{x},t)}{\partial t}$ as shown in Figure (a),

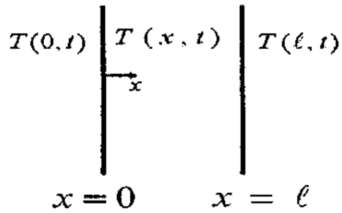


Figure (a)

where $k$, C and $\rho$ are constants with definitions expressed as follows:

$k$: conductivity coefficient ($\frac{kJ}{\text{sec}-m-k}$); C: specific heat($\frac{kJ}{kg-k}$) ; $\rho$: density ($\frac{kg}{m^3}$).

**or** Drive the 1-D wave equation of string $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$ as shown in Figure(b), where $a^2=\frac{T}{\rho}$ is a constant, $\rho$ is the mass per unit length and the tension $T$ acts tangent to the string.

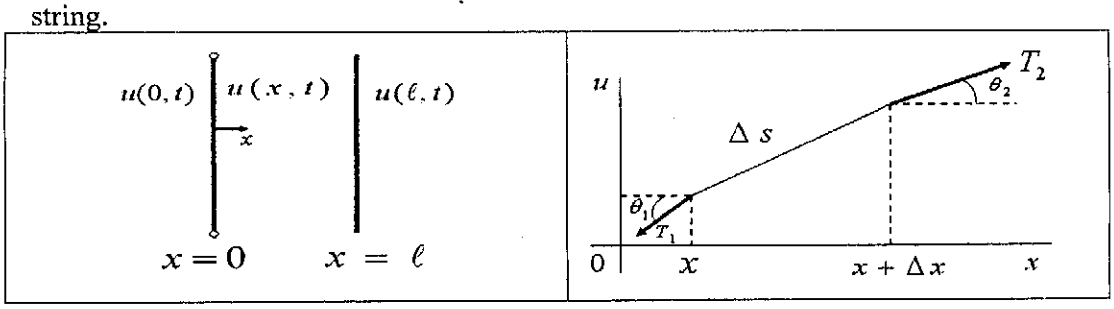


Figure (b)

5. (15%) The 1-D Heat equation is expressed as $\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}$, where the boundary conditions and initial condition are given by

$T(0, t)=\cos t,\ t>0$

$T(\ell, t)=0,\ t>0$

$T(x, 0)=0,\ 0<x<\ell$

Solve the temperature $T(x, t)$.