1. (10%) As shown in Fig. 1, consider a particle P expressed in its polar coordinates r and θ, and let $\vec{r}$, $\vec{e}_r$ and $\vec{e}_\theta$ respectively denote the position vector of the particle P and the radial and transverse unit vectors attached at P. Derive the velocity and acceleration of the particle P in terms of $\vec{e}_r$ and $\vec{e}_\theta$.

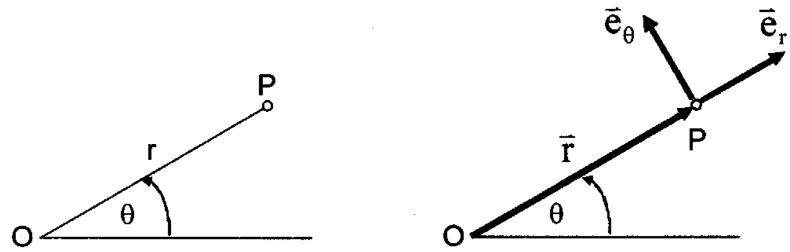

Fig. 1

2. (10%) As shown in Fig. 2, consider an inverted pendulum (with mass **m** and length ℓ) pivoted to the ground under the influence of the gravity. Assume the pendulum is lumped into a point mass at the tip with a mass-less connecting rod. Use the Newton's laws and the kinematic relations obtained in Problem 1 to verify that the governing equation of the dynamics of the pendulum is

$$\ell\ddot{\theta} = g\sin\theta$$

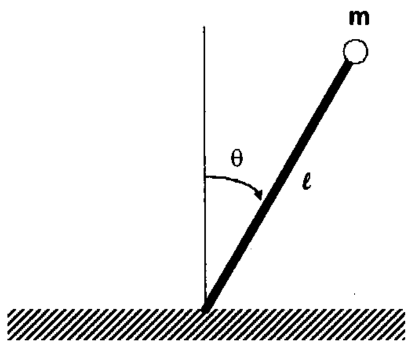

Fig. 2

3. (10%) Consider an inverted pendulum (with mass **m** and length ℓ) pivoted to a moving cart (with mass **M**) under the influence of the gravity and an external force **F** as shown below. Assume the cart moves frictionlessly and the pendulum is lumped into a point mass at the tip with a mass-less connecting rod as shown in Fig. 3. Use the Newton's laws and extend the kinematic relations obtained in Problem 1 to derive the differential equations governing the dynamics of the moving pendulum.

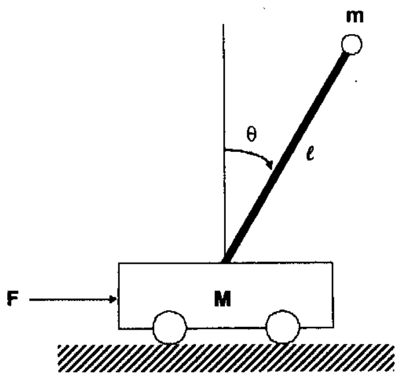

Fig. 3

4. (20%) As shown in Fig. 4, a sphere is rolling on the curve surface which has the radius equal to *R*. Pure rotation are assumed on the surface with no slip occurs. The mass of the sphere is equal to *m* and the radius is equal to *r*. If the ball is released at the position A, please answer the following questions.
   (a) (10%) The linear velocity of the center of the ball at the position B.
   (b) (10%) The perpendicular forces N at the position B when the sphere is at the position B as shown in Fig. 4

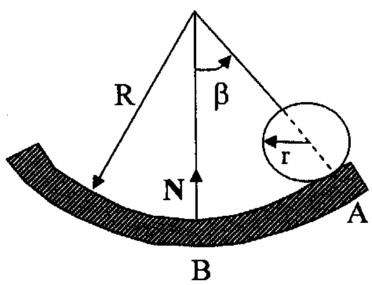

Fig. 4

5. (20%) A sphere strikes a thin bar in oblique eccentric impact as shown in Fig. 5. Assume that the bar is pinned at O and initially at rest. Please determine the velocity of the bar and sphere after impact.
Here v_1=50 m/s, θ=60^0, l=6m, l_1=4m, m_1=2Kg, m_2=4Kg, and e=0.8. e is the coefficient of restitution. m_1 and m_2 are the masses of the sphere and the bar, respectively.

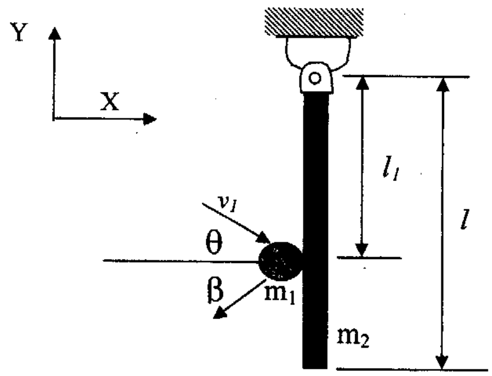

Fig. 5

6. (30%) Consider a system consisted of two rigid bodies, denoted as elements A and B, as shown in Fig. 6. Element A is a uniform U-shaped plate which can rotate freely with respect to the fixed axis L passing through points P and Q. Element B is a uniform cylinder mounted on the center of element A. It can rotate freely with respect to its center axis Z. Assume that all contacts are frictionless. Suppose that the mass centers of elements A and B coincide at point G. Suppose also that there is a body-fixed (fixed to element A) coordinate XYZ located at G. Let us denote the following notations.

   m_A, m_B: mass of elements A and B, respectively.

   I_{AX}, I_{AY}, I_{AZ} : mass moment of inertia of element A with respect to the 3 coordinate axes X, Y, and Z (which are also principal axes).

   I_{BX}, I_{BY}, I_{BZ} : mass moment of inertia of element B with respect to the 3 coordinate axes X, Y, and Z (which are also principal axes).

   $(-l, 0, h)$: coordinates of point P with respect to the body-fixed coordinate XYZ.

   $(-l, 0, h)$: coordinates of point Q with respect to the body-fixed coordinate XYZ.

   $\theta_A, \dot{\theta}_A$: angular displacement and velocity of element A with respect to the axis L.

   $\theta_B, \dot{\theta}_B$: angular displacement and velocity of element B with respect to the axis Z.

   g : gravitational acceleration.

   (a) (10%) Please express the kinetic energy and potential energy of element A in terms of the given variables.

   (b) (20%) Please express the kinetic energy and potential energy of element B in terms of the given variables.

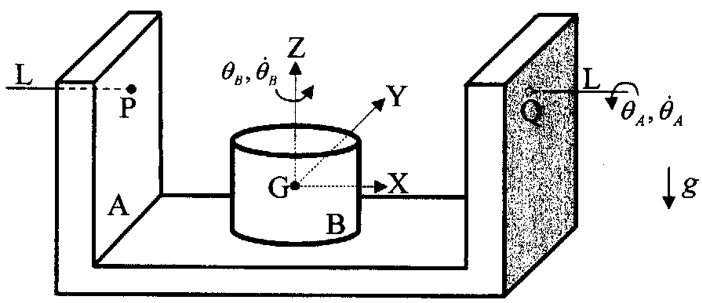

Fig. 6