1. Solve the following differential equation:

$$\frac{dv}{dt} = 1 - v^2;\ \ v(0) = 2,$$

to obtain the dependence of the variable v on the independent variable $t \in [0, \infty)$. **(10%)**

2. Solve the following integral equation:

$$\delta(t) = y(t) + \int_0^t y(\tau)\, d\tau;\ \ y(0) = 0,$$

for the variable y being a function of $t \in [0, \infty)$, where δ denotes the *Dirac delta function*. **(15%)**

3. For a 2x2 matrix

$$\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix},$$

a) Find the eigenvalues andeignevectors of this matrix **(7 %)**

b) Find the diagonalization of this matrix **(8%)**

4. Given the force vector **F** = (xy^2+xz^2, yz^2+yx^2, zx^2+zy^2)

(a) Show that **F** is a CONSERVATIVE force field. **(5%)**

(b) Find the POTENTIAL FUNCTION of the force field. **(5%)**

5. Use Green's Theorem to evaluate the line integral

$$I = \int_C y^3\, dx + (x^3 + 3xy^2)\, dy$$

Where C is the path from (0, 0) to (1, 1) along the graph of $y = x^3$ and from (1, 1) to (0, 0) along the graph of y = x. **(10%)**

6. Solve the following partial differential equation:

$x^2\frac{\partial^2 U}{\partial x\partial y}+3y^2U=0$ **(10%)**

7.

(a) Fin d the Fourier transform, $F(\alpha)$, of $f(x)=\begin{cases}3 & |x|<a\\ 0 & |x|>a\end{cases}$. **(4%)**

(b) Graph $f(x)$ and its Fourier transform $F(\alpha)$ for $a=2$. **(4%)**

(c) If $f(x)$ is an even function, show that:

$F(\alpha)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos\alpha x\,dx=?$ **(4%)**

(d) Find $\int_0^\infty \frac{\sin^2 x}{x^2}dx=?$ **(3%)**

8.

(a) Prove that $\sin(x+iy)=\sin x\cosh y+i\cos(x)\sinh(y)$ **(4%)**

(b) If z is a complex variable, $z=x+iy$, evaluate the integral $\int_{1+i}^{2+4i} z^2dz$ along the parabola $x=t, y=t^2$ where $1\le t\le 2$.

(4%)

(c) If C is a simple closed curve bounding a region having $z=a$ as interior point, find $\oint_C \frac{dz}{(z-a)^n}=?$ where $n=1, 2, 3, 4, 5$ ....

(4%)

(d) Evaluate $\int_0^{2\pi}\frac{1}{5+3\cos\theta}d\theta=?$. **(3%)**