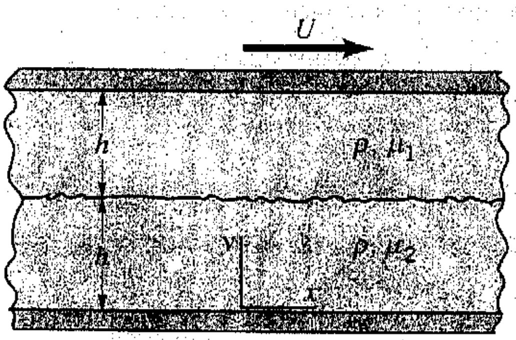
1. Two immiscible, incompressible, viscous fluids having the same densities but different viscosities are contained between two infinite, horizontal, parallel plates as shown in the following figure. The bottom plate is fixed and the upper plate moves with a constant velocity U. The motion of the fluid is caused entirely by the movement of the upper plate; that is, there is no pressure gradient in the x direction.

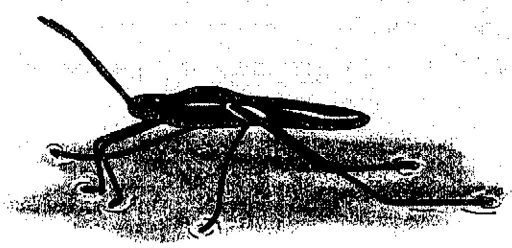
(a) Assume the laminar and steady flow. Start with the Navier-Stokes equations, and reduce them to find the proper differential equation to satisfy this problem. (5%)

(b) Please specify proper boundary conditions. (4%)

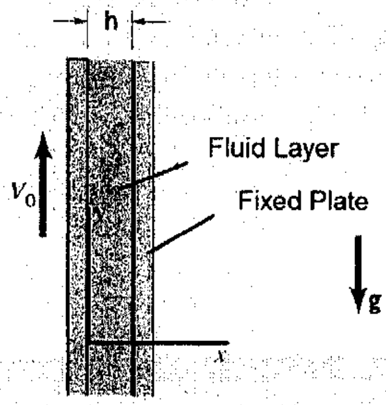
(c) Determine the velocity at the interface in terms of U, μ_1 and μ_2. (6%)



2. The water strider bug shown in the following figure is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force (7.34×10^{-2} N/m) acts vertically upwards. (10%)



3. A viscous liquid with a viscosity of μ and a density of ρ is sandwiched between a vertically upward moving pate (with a constant velocity of V_0) and a fixed plate, as shown in the following figure. Assume that the flow between the two infinite plates is steady and laminar, and the pressure gradient in the y-direction is negligible.



(a) Starting with the continuity and Navior-Stokes equations, please derive (推導) the following three equations (12%).

$$\frac{\partial P}{\partial x}=0, \quad \frac{\partial P}{\partial z}=0, \quad 0=-\rho g+\mu\frac{\partial^2 v}{\partial x^2}$$

(b) Please specify boundary conditions at x=0 and x=h for the y-direction velocity (6%).

(c) Please determine the y-direction velocity distribution (7%).

Given:

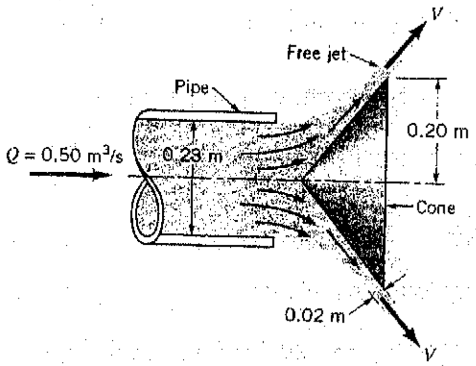
Continuity equation:

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\rho\vec{v}=0$$

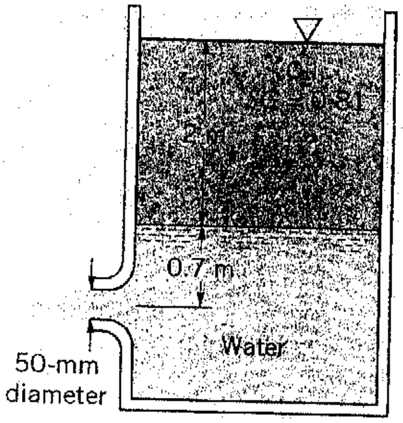
Navior-Stokes Equations:

$$\rho\frac{D\vec{v}}{Dt}=+\rho\vec{g}-\nabla P+\mu\nabla^2\vec{u}$$

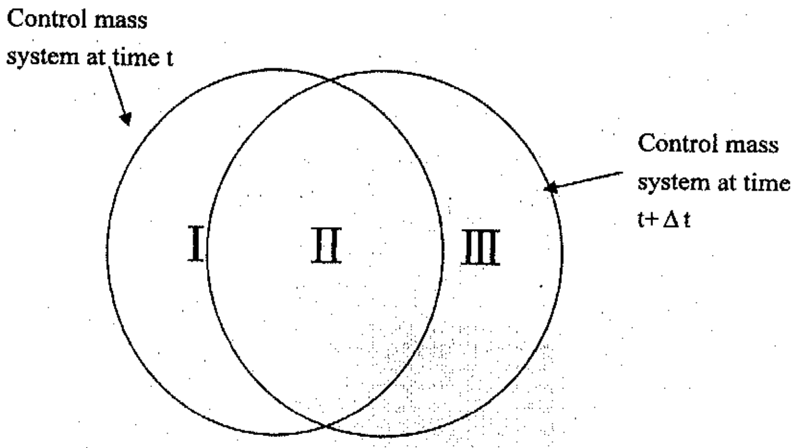
4. A conical plug is used to regulate the air flow from the pipe shown in the figure. The air leaves the edge of the cone with a uniform thickness of 0.02 m. If viscous effects are negligible and the flowrate is 0.50 m^3/s, determine the pressure within the pipe. Assume the density of air is 1.23 kg/m^3. (15%)



5. If viscous effects are neglected and the tank is large, determine the flow rate from tank shown in the following figure. (10%)



6.



(a) Which close regions of I、II and III is the control volume? Please account for your reason. (5%)

(b) Given N is the extensive property of the fluid，please compute the rate of change of N with respect to time for your set the control colure system than to prove that：$\frac{DN}{Dt}=\oiint_{C.S} n\rho\vec{V}\bullet d\vec{A}+\frac{\partial}{\partial t}\iiint_{CV} n\rho d\forall$，where $n=\frac{N}{mass}$，$\forall$ is the volume of fluid on control volume，$d\vec{A}$ is the control surface of control volume) (10%)

(c) Assume N is mass of fluid. Please set the differential form of continuity equation. (10%)