1. (40%) In the Fig. 1, G stands for the open-loop plant with its transfer function being

$$G(s) \equiv \frac{6s^2 + 6s + 5}{s^3},$$

and r for the referenced command that is a step input to the closed-loop system $G_{c\ell}$ and is being tracked by the measurement output y.

(a) (2%) What is the transfer function of the closed-loop system $G_{c\ell}$, $G_{c\ell}(s)$?

(b) (2%) What are the poles of $G_{c\ell}(s)$?

(c) (6%) Is the $G_{c\ell}$ asymptotically stable? Is it bounded-input-bounded-output stable? Is it exponentially stable?

(d) (5%) What is the steady-state error e_{ss} of $G_{c\ell}$?

(e) (5%) Estimate the rising time t_r of $G_{c\ell}$.

(f) (5%) Estimate the overshoot M_p of $G_{c\ell}$.

(g) (5%) Estimate the settling time t_s of $G_{c\ell}$.

(h) (5%) Is $G_{c\ell}(s)$ of minimum phase?

(i) (5%) Estimate the peak value of the Bode-magnitude plot of $G_{c\ell}(s)$.

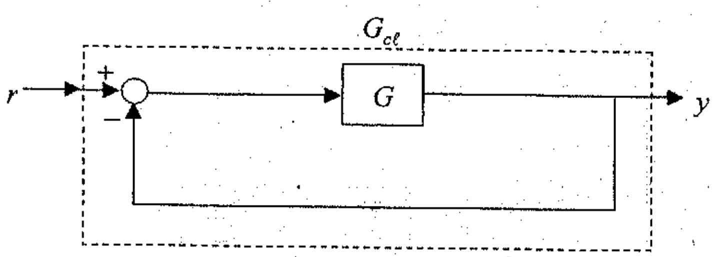

Fig. 1 Dynamic response of feedback system

2. (30%) Given a feedback system as shown in the figure below, where r is the reference input, y is the output, e is the error. K is a constant gain.

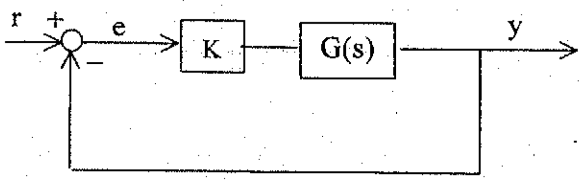

Fig. 2

The plant G(s) is given as:

$$G(s) = \frac{20}{s(s+2)(s+10)}$$

(a) (5%) Please determine the frequency ω_{GM} **analytically** where the phase of $G(j\omega_{GM})$ is equal to -180^0

(b) (5%) Find the stable range of K for the closed loop system between r and y using the information obtained. What is the gain margin?

(c) (5%) Plot the Bode Diagram of G(s)

(d) (5%) Find the ***approximate*** value of the crossover frequency ω_c using the Bode Diagram in (c).

(e) (5%) Use the ω_c obtained in (d) to calculate the phase margin of the system.

(f) (5%) Explain why any K smaller than 0 will be unstable using Nyquist criterion.

3. (30%) The dynamics of an inverted pendulum system can be approximately described by

$$\ddot{y} - 4y = 3u$$

where y is the inclined angle of the pendulum, and u is the input force.

(a) (5%) With u as the input and y as the output, please determine the transfer function $G(s)$ of the pendulum system.

(b) (25%) Consider now a control system with plant $G(s)$ (the pendulum system) and controller $C(s)$, as shown in Fig. 3. The goal is to stabilize the pendulum system, i.e., to make the overall closed system stable. For each of the following controllers $C(s)$, please use the method of root locus to determine if the goal can be achieved.

(i) Proportional controller (P-control): $C(s) = k, \quad k \geq 0$;

(ii) Proportional-derivative controller (PD-control): $C(s) = k(s+10), \quad k \geq 0$;

(iii) Proportional-integral controller (PI-control): $C(s) = \dfrac{k(s+0.1)}{s}, \quad k \geq 0$

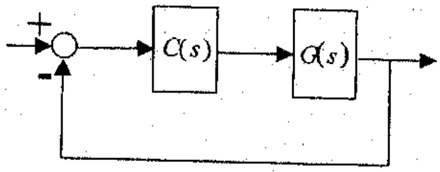

Fig. 3