

1. (25%) The variable  $y: \mathfrak{R} \rightarrow \mathfrak{R}$  is governed by  $\dot{y} + y = \dot{u}$ ,  $y(0) = 0$ , with the input  $u: \mathfrak{R} \rightarrow \mathfrak{R}$ .
  - (a) (10%) Please solve the impulse response of  $y$ , that is,  $u(t) = \delta(t)$ ;
  - (b) (7%) Based on the impulse response obtained in (a), find the step response of  $y$ ;
  - (c) (8%) Solve the frequency response of  $y$ , i.e. the forcing response of  $y$  as  $u(t) = \cos \omega t$ .

2. (15%) Consider the following feedback system shown in Figure 1.

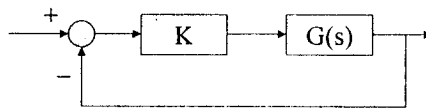


Figure 1.

Use the root locus method to discuss the correctness (true or false) of the following statements. **No point will be given to discussions that do not use the root locus method.**

- (a) (5%) If  $G(s)$  is a stable first order system, the closed loop system is stable for all  $K > 0$ .
- (b) (5%) When  $G(s)$  is a stable second order system with a zero in the right half plane, the closed loop system will eventually become unstable as  $K$  goes to  $+\infty$ .
- (c) (5%) Express  $G(s) = N(s)/D(s)$  where  $N(s)$  and  $D(s)$  are polynomials. If the degree of  $D(s)$  is greater than that of  $N(s)$  by at least 3, the closed loop system will eventually become unstable as  $K$  goes to  $+\infty$ .

3. (15%) Consider the following two closed-loop systems (02) where

$$G_1(s) = \frac{1}{s(s+1)(s+100)} \quad \text{and} \quad G_2(s) = \frac{1}{100s(s+1)}$$



Figure 2.

- (a) (5%) Identify the segments on the real axis which are parts of the root locuses of  $G_1(s)$  and  $G_2(s)$ , respectively.
- (b) (10%) Explain why the behaviors of the root locuses around the breakaway point of these two closed-loop systems are similar.