國立中正大學九十四學年度碩士班招生考試試題

系所別：機械工程學系(甲組、乙組、丙組)　　　　科目：工程數學

1. (a) Find the eigenvalues and the corresponding eigenvectors of the following matrix. (15%)

$$\mathbf{A} = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

   (b) Find rank of matrix $\mathbf{A}$. Is matrix $\mathbf{A}$ a singular matrix? (10%)

2. For a linear partial differential equation of second-order:

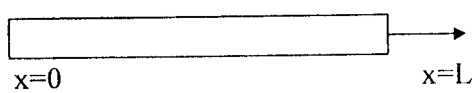
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial T}{\partial t},$$

   where $\alpha^2 \equiv \dfrac{k}{\rho\, C_P}$,

   and k is the thermal conductivity, ρ is the density of a bar, C_P is the specific heat at constant pressure

   (a) Define the differential equations for the parabolic and elliptic types. (5%)

   (b) Find the solution of T(x,t), given boundary conditions as T(0,t)=2, T(L,t)=1, and initial condition as T(x,0)=f(x). (10%)



   (c) Find the temperature increasing rate at x=0。　　　(10%)

3. The basis Φ is assigned by $\Phi \equiv \{\phi_n : [0,\pi] \to \Re \mid \phi_n(x) = \sin nx,\ n = 1,2,\cdots,\infty\}$. It is known that any function $f:[0,\pi] \to \Re$ can be linearly spanned by Φ, i.e. $f = a_1\phi_1 + a_2\phi_2 + \cdots$, where a_i is the coordinate with respect to ϕ_i. The inner product $<\bullet,\bullet>$ over Φ is defined by $<\phi_i,\phi_j> \equiv \int_0^\pi \phi_i(x)\phi_j(x)\,dx$. ϕ_i and ϕ_j are mutually orthogonal if $<\phi_i,\phi_j> = 0$.

   (a) Prove that any two members of Φ are mutually orthogonal. (7%)

   (b) Let a function $f:[0,\pi] \to \Re$ be orthogonal to all members of Φ. Prove that $f \equiv 0$. (8%)

   (c) If a function $f:[0,\pi] \to \Re$ is defined by $f(x) = \begin{cases} 1; & 0 \le x \le \dfrac{\pi}{2} \\ -1; & \dfrac{\pi}{2} < x \le \pi \end{cases}$, then calculate the coordinate a_i with respect to ϕ_i for $i = 1,2,\cdots,\infty$. (10%)

4. (a) Find a general solution to the ordinary differential equation. (10%)

$$y''' - 4y'' + 4y' - y = 4$$

   (b) Solve the initial value problem. (15%)

$$y'' + 4y' + 4y = e^{-2t},\ y(0) = 1,\ y'(0) = 0$$